Mathematics
Higher Level
Paper 2

Name

Date: _____

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 14 pages

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, for example if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A (57 marks)

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 6]

A study is conducted to compare the monthly e-commerce sales of nine different online stores to their monthly online advertising costs. The table below shows, in units of \$1000, the monthly e-commerce sales (y) of each online store and their monthly online advertising costs (x).

The relationship between the monthly e-commerce sales and the monthly online advertising costs can be modelled by the regression line with equation $y = ax + b$.

Online Advertising Costs (x)	1.4	1.7	2.3	1.1	4.7	2.2	2.9	3.8	1.9
E-Commerce Sales (y)	343	371	587	320	921	492	646	835	413

(a) (i) Find Pearson's product moment correlation coefficient, r.

(ii) Write down the value of a and the value of b. [3]

One of these nine online stores decides to increase their budget for monthly online advertising costs by \$500.

(b) Based on the given data, determine how the store's monthly e-commerce sales could be expected to alter. [2]

An online store separate from the study has monthly online advertising costs of \$7000.

(c) Comment on the appropriateness of using your regression line to predict the monthly e-commerce sales of this separate online store. [1]

(This question continues on the following page)

(Question 1 continued)

2. [Maximum mark: 6]

Triangle FGH has $FG = 8$ cm, $GH = 9$ cm and area 24 cm^2.

(a) Find $\sin \hat{G}$. [2]

(b) Hence, find the two possible values of FH, giving your answers correct to two decimal places. [4]

3. [Maximum mark: 6]

The sum of the first n terms of a series is given by

$$S_n = 3n^2 + n, \ n \in \mathbb{Z}^+$$

(a) Find the first three terms of the series. [3]

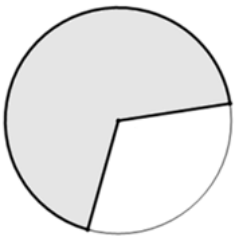
(b) Find an expression for the n^{th} term of the series, giving your answer in terms of n. [3]

4. [Maximum mark: 6]

Using the substitution $2x = \sin\theta$, or otherwise, find $\int\left(\sqrt{1-4x^2}\right)dx$.

5. [Maximum mark: 6]

In the figure below, the shaded sector has a perimeter that is equal to the circumference of the circle. The location of a point inside the circle is chosen at random. Find the probability that the randomly chosen point is located inside the shaded sector.



6. [Maximum mark: 7]

The line L is given by the parametric equations $x = 1 - \lambda,\ y = 2 - 3\lambda,\ z = 2$. Find the coordinates of the point on L which is nearest to the origin.

7. [Maximum mark: 6]

Consider the function $f(x) = \ln(1-x^2)$.

(a) (i) Determine the domain of $f(x)$.

(ii) Find the first three terms in the Maclaurin series for $f(x)$. [3]

(b) Hence, show that the exact value of $\sum_{n=1}^{\infty} \frac{2^{-2n}}{n}$ is $\ln\left(\frac{4}{3}\right)$. [3]

8. [Maximum mark: 8]

If α and β are the roots of the equation $2x^2+6x-5=0$, find a quadratic equation with integer coefficients whose roots are:

(a) 2α, 2β [4]

(b) $\frac{1}{\alpha+1}, \frac{1}{\beta+1}$ [4]

9. [Maximum mark: 6]

There are 25 marbles in a bag. Some of them yellow and the rest are blue. Two marbles are simultaneously selected at random. Given that the probability of selecting two marbles of the same colour is equal to the probability of selecting two marbles of different colour, determine the number of yellow marbles in the bag.

Do **not** write solutions on this page.

Section B (53 marks)

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 21]

Consider the quadratic function defined by $f(x) = x^2 - 7x + 13$ with domain $x \geq \frac{7}{2}$.

(a) Sketch the graph of f. [2]

(b) State the range of f. [2]

(c) Find the inverse of function of f, that is find $f^{-1}(x)$. [5]

(d) Write down the domain and range of $f^{-1}(x)$. [2]

(e) Find all values of x such that $f(x) = f^{-1}(x)$. [3]

Consider the function $g(x) = \frac{1}{x-1}$.

(f) Sketch the graph of $g(f(x))$, clearly indicating any asymptotes. [3]

(g) Determine the domain of $g(f(x))$. [2]

(h) Determine the range of $g(f(x))$. [2]

Do **not** write solutions on this page.

11. [Maximum mark: 7]

It has been determined that the volume of fluid in a bottle of olive oil filled by a robotic dispenser in a factory is normally distributed with a mean of 748 ml and a standard deviation of 2.4 ml.

(a) Show that the probability that a randomly selected bottle of olive oil from the factory contains more than 750 ml is approximately 0.202 [1]

(b) The amount of olive oil is measured for each bottle in a random sample of 12 bottles. Find the probability that exactly 4 of them contain more than 750 ml. [3]

(c) Find the minimum number of bottles that would need to be sampled so that the probability of getting at least one bottle containing more than 750 ml of olive oil is greater than 0.98. [3]

The same factory produces bags of flour, such that the weight, A grams, of flour in a bag is normally distributed with mean μ grams and standard deviation σ grams.

(d) Given that $\mathrm{P}(A < 850) = 0.09$ and $\mathrm{P}(A < 900) = 0.97$, find the value of μ and the value of σ. [6]

A bag of flour is deemed to be insufficient if it contains less than m grams of flour. It is known that, out of every 1000 bags, 15 are deemed insufficient.

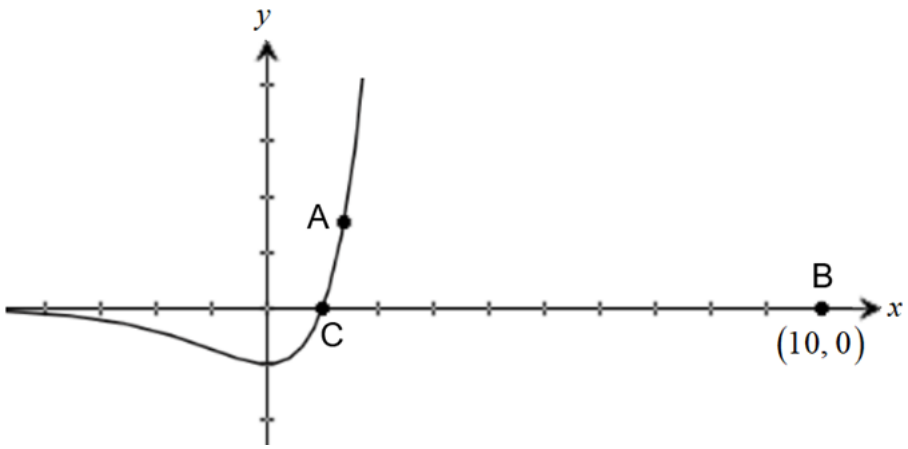
(e) Calculate the value of m. [3]

[exam continues on next page]

Do **not** write solutions on this page.

12. [Maximum mark: 16]

Consider the graph of the function $h(x) = (x-1)e^x$ shown below.



(a) Point C is the x-intercept of h. Find the equation of the line tangent to h at C. [4]

(b) Point A, with x-coordinate of a, lies on the graph h. Show that the equation, in terms of x and a, of the line that is tangent to h at point A is $y = ae^a x + e^a(-a^2 + a - 1)$. [3]

(c) Find the equation, in terms of x and a, of the line that is **normal** to h at point A. [4]

(d) Calculate the x-coordinate of the point on h that is closest to B. [5]